1 Appendix to footnote 5

Below we list three other examples of the connection between equity ownership and knowledge transfer.

• Groupe Aeroplan Inc. announced in 2010 that it would acquire a stake of less than 20% of Grupo AeroMexico’s frequent flyer program, Club Premier. Aeroplan’s Chief Executive Officer said that Club Premier would benefit from Aeroplan’s know-how and develop the necessary skill sets critical to its successful transformation into a profitable coalition program. Groupe Aeroplan and Grupo Aeromexico compete with each other in the air travel market by offering incentives to frequent travellers through frequent flyer programs.

• Harvey World Travel announced in 2004 its plan to take an initial 11% equity holding in Webjet, an internet travel business specialist. Webjet’s Managing Director, David Clarke, said that the arrangement would provide Webjet with a strategic development partner which would enhance Webjet’s ability to capitalize on opportunities in a rapidly changing travel market in the Australian region. The two firms compete in the market of travel services such as hotel and flight booking.

• USEN announced in August 2015 that it was going to form an equity strategic alliance with CAN System in which USEN, after acquiring 10% stake in CAN System, will provide its know how regarding digitization of music broadcasting. USEN and CAN compete in the market of music provision to small businesses such as restaurants and individuals.

Details on these examples are available, respectively, from:
http://m.theglobeandmail.com/globe-investor/aeroplan-buying-stake-in-aeromexico-loyalty-program/article1547079/?service=mobile,

2 Appendix to Section 4: Industry average cost and PEO

In the second paragraph after the proof of Proposition 4, we stated that “Even when the endogenously determined level of PEO reduces consumer surplus, we find that PEO always reduces the industry average cost given that a larger fraction of the industry output is produced at the lower cost $c – x.$” Below we prove that statement.

Let $AC(\theta, k, n)$ denote the industry average cost for a given $(\theta, k, n)$. We have that

$$AC(\theta, k, n) = \frac{\sum_{i=1}^{n+2} c_i q_i^*(\theta, k, n)}{Q^*(\theta, k, n)}$$
Claim A: Firm 1’s equilibrium profit in the stage 3 subgame, $\pi^*_1(\theta, k, n)$, is strictly convex in $k$ (i.e., $\frac{d^2 \pi^*_1(\theta, k, n)}{dk^2} > 0$) and attains its maximum at $k = 0$ or $k = 1$.

**Proof:** Using (3) - (7) for $P(Q) = a - dQ$, we can write $\pi^*_1(\theta, k, n)$ as

$$\pi^*_1(\theta, k, n) = d(q^*_1(\theta, k, n))^2 + \theta q^*_1(\theta, k, n)q^*_2(\theta, k, n) + \theta q^*_2(\theta, k, n)^2).$$

Differentiating $\pi^*_1(\theta, k, n)$ twice and simplifying subsequently we get

$$\frac{d^2 \pi^*_1(\theta, k, n)}{dk^2} = 2d(1-\theta)(\frac{dq^*_1(\theta, k, n)}{dk})^2 + 2d\theta[\frac{dq^*_1(\theta, k, n)}{dk} + \frac{dq^*_2(\theta, k, n)}{dk} + \frac{dq^*_1(\theta, k, n) dq^*_2(\theta, k, n)}{dk} > 0$$
where the strict inequality follows from noting that \( d > 0 \), \( \frac{dq_i^1(\theta,k,n)}{dk} = \frac{-(1+2\theta)x}{d(1-\theta)} < 0 \) and \( \frac{dq_i^2(\theta,k,n)}{dk} = \frac{3x}{d(1-\theta)} > 0 \).

Now we turn to prove the second part of the claim. Since \( k \) lies in a compact interval \([0,1]\) and \( \pi_i^*(\theta,k,n) \) is continuous in \( k \) it is immediate that \( \pi_i^*(\theta,k,n) \) attains its maximum for some \( k = k^* \in [0,1] \). Suppose \( k^* \in (0,1) \). Then (a) \( \frac{d\pi_i^*(\theta,k,n)}{dk} = 0 \) and (b) \( \frac{d\pi_i^*(\theta,k,n)^2}{dk^2} < 0 \) must hold. However, (b) cannot hold which imply either \( k^* = 0 \) or \( k^* = 1 \). \( Q.E.D. \)

Claim A implies that firm 1 either transfers its knowledge in full \((k^* = 1)\) or transfers no knowledge at all \((k^* = 0)\).

### 3.2 Partial cross ownership

Corresponding to the discussion in the last two paragraphs of Section 4, here we present an outline of the analysis of the model’s variant in which firm 2 can also hold ownership in firm 1’s equity. Let \( \psi \) \( (0 \leq \psi \leq 1) \) denote the level of firm 2’s ownership in firm 1’s equity. If \( \theta \in (\frac{1}{2},1] \) and \( \psi \in [0,\frac{1}{2}] \), firm 1 chooses \( q_1 \) and \( q_2 \). If \( \theta \in [\frac{1}{2},1] \) and \( \psi \in (\frac{1}{2},1] \), firm 2 chooses \( q_1 \) and \( q_2 \) and also makes a decision concerning knowledge transfer. As in the analysis of the original model, for expositional simplicity we assume that, if firm \( i \) \((i = 1 \text{ or } 2)\) is indifferent between shutting down and not shutting down firm 2, firm \( i \) chooses to shut down firm 2. Then, in either of the two cases mentioned above, firm 2 is shut down in the equilibrium and the equilibrium outcome is the same as the one of duopoly consisting of firms 1 and 3. Also, the equilibrium outcome under \( \theta \in (\frac{1}{2},1] \) and \( \psi \in (\frac{1}{2},1] \) is equivalent to the equilibrium outcome under \( \theta \in [\frac{1}{2},1] \) and \( \psi \in [0,\frac{1}{2}] \). Given this, in what follows we consider the case of \( \theta \in [\frac{1}{2},1] \) and \( \psi \in [0,\frac{1}{2}] \), in which firm 1 chooses \( q_1 \) and makes a decision concerning knowledge transfer, and firm 2 chooses \( q_2 \).

We assume that \( x < \min\{\frac{a-c}{2},c\} \) and \( c > \frac{a}{3} \) hold as in the linear demand model outlined in Section 4. Let \( (\theta,\psi,k,n) \) \((k \in \{0,1\})\) be given. Through a standard analysis of Cournot competition we find that the equilibrium is unique for any given \( (\theta,\psi,k,n) \), where \( q_i^*(\theta,\psi,k,n) \) and \( \pi_i^*(\theta,\psi,k,n) \) \((i = 1,2,...,n+2)\) respectively denote each firm \( i \)’s equilibrium quantity and profit. Let \( \pi_{12}^*(\theta,\psi,k,n) \equiv \pi_1^*(\theta,\psi,k,n) + \pi_2^*(\theta,\psi,k,n) \). We find that \( q_i^*(\theta,\psi,1,n) > 0 \) for \( i = 1,2,...,n+2 \) and \( q_i^*(\theta,\psi,0,n) > 0 \) for \( i = 1,3,...,n+2 \), while \( q_2^*(\theta,\psi,0,n) > 0 \) if \( \psi < \frac{1}{1-\theta} \frac{a-c-x}{a-c+(n+1)x} \) and \( q_2^*(\theta,\psi,0,n) = 0 \) otherwise.

Suppose that firms 1 and 2 choose \( (\theta,\psi) = (\theta^*,\psi^*) \) \((\theta^* \in [0,\frac{1}{2}] \text{ and } \psi^* \in [0,\frac{1}{2}]\) at Stage 1 and knowledge is transferred at Stage 2 in the equilibrium. We then find that \( \psi^* = 0 \) must hold. To see this, suppose \( \psi^* > 0 \) where \( \psi^* < \frac{1}{1-\theta^*} \frac{a-c-x}{a-c+(n+1)x} \) holds. Define \( \Delta(\theta,\psi,n) \equiv \pi_1^*(\theta,\psi,1,n) - \pi_1^*(\theta,\psi,0,n) \), where \( \Delta(0,\psi,n) = -\frac{\pi_1(1-\psi^*)[2(\psi^*)^2+(2n+1)\psi^*-(n+1)x]}{d(\psi^*)^2} < 0 \) holds. We also have that \( \frac{\partial}{\partial \theta} \pi_{12}^*(\theta,\psi^*,1,n) = -\frac{(n+1+\theta^*)[a-c+(n+1)x]}{d(n+3-\theta^*)^2} < 0 \) for all \( \theta \in [0,\theta^*] \). This implies \( \Delta(\theta^*,\psi^*) = 0 \). It can be shown that \( \Delta(\theta,0,n) > \Delta(\theta^*,\psi^*,n) = 0 \). We also have that \( \frac{\partial}{\partial \psi} \pi_{12}^*(\theta^*,\psi,1,n) = -\frac{(n+1+\theta^*)[a-c+(n+1)x]}{d(n+3-\theta^*)^2} < 0 \) for all \( \psi \in [0,\psi^*] \). This implies that firms 1 and 2 would be strictly
better off by choosing \((\theta, \psi) = (\theta^*, 0)\) over \((\theta, \psi) = (\theta^*, \psi^*)\) at Stage 1, because \(\Delta(\theta^*, 0) > 0\) implies that knowledge is transferred in the subsequent equilibrium under \((\theta, \psi) = (\theta^*, 0)\) where \(\pi_{12}^*(\theta^*, 0, 1) > \pi_{12}^*(\theta^*, \psi^*, 1)\) holds. This is a contradiction. Similarly, we reach a contradiction by supposing \(\psi^* > 0\) where \(\psi^* \geq (1 - \theta^*)a^{-c} - \frac{x}{a-c+(n+1)x}\) holds. The analysis of the original model then implies that firms 1 and 2 choose \((\theta, \psi) = (\hat{\theta}(x, n), 0)\) in the equilibrium in which knowledge is transferred at Stage 2, where \(\hat{\theta}(x, n)\) is as defined in the text.

Next suppose that firms 1 and 2 choose \((\theta, \psi) = (\theta^*, \psi^*)\) \((\theta^* \in [0, \frac{1}{2}]\) and \(\psi^* \in [0, \frac{1}{2}]\)) at Stage 1 and knowledge is transferred at Stage 2 in the equilibrium. Since firm 2 is cost inefficient without knowledge transfer, \(\psi^* > 0\) holds in the equilibrium. We also find that \(\theta^* = 0\) must hold. To see this, suppose \(\theta^* > 0\) and \(\psi^* > 0\) hold. Consider \(\theta^{**} \equiv \theta^* - \Phi\) and \(\psi^{**} \equiv \psi^* + \Phi \frac{a-c-(1-\psi^*)x}{a-c+(n+2-\theta^*)x}\), where \(\Phi(>0)\) is small enough such that \(\theta^{**} > 0\) and \(\psi^{**} > 0\) hold. We find that the equilibrium price and the equilibrium joint output of firms 1 and 2 are same under \((\theta, \psi) = (\theta^*, \psi^*)\) and \((\theta, \psi) = (\theta^{**}, \psi^{**})\). This means that firms 1 and 2 will be strictly better off by choosing \((\theta, \psi) = (\theta^{**}, \psi^{**})\) over \((\theta, \psi) = (\theta^*, \psi^*)\), implying that \(\theta^* > 0\) cannot hold in the equilibrium. Given this, we find that the equilibrium of this variant of the model is as described in the second last paragraph of Section 4 of the text.

4 Appendix to Section 5: PEO vs. the merger/non-merger dichotomy

Consider the model with linear demand and suppose merger and independence were the only options for firms 1 and 2 at Stage 1. In what follows, this model with only two options is referred to as the merger/non-merger dichotomy model, and our original model is referred to as the PEO model to make the distinction clear. In the dichotomy model, we find that there exists a threshold value \(\hat{x}(n) \equiv \frac{n^2+2n-1}{3n^2+12n+11}(a - c) \in (0, \bar{x})\) such that firms 1 and 2 choose to merge if \(x > \hat{x}(n)\) and remain independent if \(x < \hat{x}(n)\). We also find that there exists a threshold value \(\bar{x}_{TS}(n) \equiv \frac{2n+5}{2n^2+12n+17}(a - c) \in (0, \bar{x})\) such that equilibrium total surplus is higher under merger than under non-merger if and only if \(x > \bar{x}_{TS}(n)\). We find that \(\hat{x}(n) < \bar{x}_{TS}(n)\) if \(n = 1\) or 2, and \(\bar{x}_{TS}(n) < \hat{x}(n)\) if \(n \geq 3\).

When PEO is introduced as an option, firms 1 and 2 choose \(\theta = \hat{\theta}(x, n) \in (0, \frac{1}{2}]\) for all \(x \in (x_{min}(n), x_{max}(n)]\) as shown in Proposition L1, whereas they choose either \(\theta = 0\) or \(\theta = 1\) in the dichotomy model. The choice of PEO \(\theta = \hat{\theta}(x, n)\) increases not only the alliance partners’ joint profit but also the total surplus, except for the case of \(n = 1\) and \(x \in (x_{min}(1), x_{TS}')\).

In order to compare policy implications of the dichotomy model and those of the PEO model, we compare equilibrium outcomes of the two models with and without the antitrust authority, where the authority maximizes equilibrium total surplus by announcing a maximum
Comparison of equilibrium outcomes: PEO model and dichotomy model

<table>
<thead>
<tr>
<th>$n = 1$</th>
<th>Range of $x$</th>
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<th>Dichotomy model</th>
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<tr>
<td>$(\hat{x}(1), x_{\text{max}}(1))$</td>
<td>No Antitrust</td>
<td>PEO</td>
<td>Merger</td>
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<td>Antitrust</td>
<td>PEO</td>
<td>Non-merger</td>
</tr>
<tr>
<td>$(x'_{TS}, \hat{x}(1))$</td>
<td>No Antitrust</td>
<td>PEO</td>
<td>Non-merger</td>
</tr>
<tr>
<td></td>
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<td>PEO</td>
<td>Non-merger</td>
</tr>
<tr>
<td>$(x_{\text{min}}(1), x'_{TS})$</td>
<td>No Antitrust</td>
<td>PEO</td>
<td>Non-merger</td>
</tr>
<tr>
<td></td>
<td>Antitrust</td>
<td>Non-PEO</td>
<td>Non-merger</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th>Dichotomy model</th>
</tr>
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<td>PEO</td>
<td>Merger</td>
</tr>
<tr>
<td></td>
<td>Antitrust</td>
<td>PEO</td>
<td>Merger</td>
</tr>
<tr>
<td>$(\hat{x}(2), \hat{x}_{TS}(2))$</td>
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<td>PEO</td>
<td>Merger</td>
</tr>
<tr>
<td></td>
<td>Antitrust</td>
<td>PEO</td>
<td>Non-merger</td>
</tr>
<tr>
<td>$(x_{\text{min}}(2), \hat{x}(2))$</td>
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<td>PEO</td>
<td>Non-merger</td>
</tr>
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<td></td>
<td>Antitrust</td>
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<td>Non-merger</td>
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<table>
<thead>
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<th>Dichotomy model</th>
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<tbody>
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<td>PEO</td>
<td>Merger</td>
</tr>
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<td></td>
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<td>Merger</td>
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<tr>
<td>$(\hat{x}_{TS}(3), \hat{x}(3))$</td>
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<tr>
<td></td>
<td>Antitrust</td>
<td>PEO</td>
<td>Non-merger</td>
</tr>
</tbody>
</table>

Table 1

permissible level of PEO at Stage 0. Table 1 presents the comparison of equilibrium outcomes for $x \in (x_{\text{min}}(n), x_{\text{max}}(n)]$.

We find that PEO can reconcile conflict of interests between alliance partners and the antitrust authority that may arise under the merger/non-merger dichotomy. Suppose $n = 1$ or 2 so that $\hat{x}(n) < \hat{x}_{TS}(n)$, and suppose $x \in (\hat{x}(n), \hat{x}_{TS}(n))$. Then firms 1 and 2 would choose to merge under the dichotomy model in the absence of the antitrust authority. In the presence of the authority, however, the authority would prohibit the merger because the merger reduces total surplus. This conflict of interests between the firms and the authority can be resolved by introducing PEO as an option. Consider the $n = 2$ case. We find that $x_{\text{min}}(2) < \hat{x}(2) < \hat{x}_{TS}(2) < x_{\text{max}}(2)$ holds, implying that the following property holds for all $x \in (\hat{x}(2), \hat{x}_{TS}(2))$: Firms 1 and 2 would choose $\theta = \hat{\theta}(x, 2)$ in the absence of the authority, and the authority would impose no restrictions on PEO. Hence there is no conflict between the firms and the authority, and the
equilibrium joint profit of firms 1 and 2 and the equilibrium total surplus are both higher in the PEO model than in the dichotomy model. When \( n = 1 \), analogous property holds when \( x \in (\hat{x}(1), \alpha_{max}(1)) \), where we find that \( x_{min}(1) < \tilde{x}(1) < x_{max}(1) < \alpha_{TS}(1) \) holds.

Now suppose \( n \geq 3 \) so that \( \alpha_{TS}(n) < \hat{x}(n) \), and suppose \( x \in (\alpha_{TS}(n), \hat{x}(n)) \). Then firms 1 and 2 would choose to remain independent under the dichotomy model in the absence of the authority. The authority, however, would prefer they merge because the merger increases total surplus. This conflict of interests can be resolved by introducing PEO as an option. Consider the \( n = 3 \) case. We find that \( x_{min}(3) < \alpha_{TS}(3) < \hat{x}(3) < x_{max}(3) \) holds. This means that, in the PEO model, firms 1 and 2 would choose \( \theta = \hat{\theta}(x, 3) \) in the absence of the authority, and the authority would impose no restrictions on PEO for all \( x \in (\alpha_{TS}(n), \hat{x}(n)) \). The equilibrium joint profit of firms 1 and 2 and the equilibrium total surplus are both higher in the PEO model than in the dichotomy model.

We also find that, when \( n = 1 \) and \( x \in (x_{min}(1), x_{TS}) \), the introduction of PEO creates a conflict of interests that is buried under the merger/non merger dichotomy. In the PEO model, firms 1 and 2 would choose \( \theta = \hat{\theta}(x, 1) \) if \( x \in (x_{min}(1), x_{TS}) \), but the authority would prohibit PEO because total surplus is higher under \( \theta = 0 \) than under \( \theta = \hat{\theta}(x, 1) \). This conflict of interests is buried in the dichotomy model because firms 1 and 2 would choose to remain independent and the authority would also prefer independence to a merger.

\section{Some details of the proof of Proposition L1}

In the proof of Proposition L1, we stated, “We find that \( g1(\tilde{x}(0, n), n) < 0 \), \( g1(\tilde{x}(\frac{1}{2}, n), n) > 0 \), and \( \frac{\partial}{\partial \theta} g1(\tilde{x}(\theta, n), n) > 0 \) for all \( \theta \in [0, \frac{1}{2}] \) and \( n \geq 1 \).” Below we present some details on these results.

- We find that \( g1(\tilde{x}(\theta, n), n) = \frac{1-\theta}{\theta(n+3)^2(n+3-\theta)^2(2n+3-(n+1)\theta)^2} f1(\theta, n)(a-c)^2 \), where \( f1(\theta, n) \equiv (n^2 + 4n + 5)\theta^5 + (n^4 + 4n^3 - 7n^2 - 52n - 64)\theta^4 + (-2n^5 - 19n^4 - 54n^3 + 9n^2 + 266n + 310)\theta^3 + (n^6 + 14n^5 + 73n^4 + 140n^3 - 106n^2 - 736n - 716)\theta^2 + (10n^4 + 124n^3 + 559n^2 + 1090n + 777)\theta - (4n^4 + 44n^3 + 180n^2 + 324n + 216) \).

- We have that \( f1(0, n) < 0 \) and \( f1(\frac{1}{2}, n) > 0 \). Also, \( \frac{\partial}{\partial \theta} f1(\theta, n) = 2\theta n^6 + (-6\theta^2 + 28\theta)n^5 + (4\theta^3 - 57\theta^2 + 146\theta + 10)n^4 + (16\theta^3 - 162\theta^2 + 280\theta + 124)n^3 + (5\theta^4 - 28\theta^3 + 27\theta^2 - 212\theta + 559)n^2 + (20\theta^4 - 208\theta^3 + 798\theta^2 - 1472\theta + 1090)n + 25\theta^4 - 256\theta^3 + 930\theta^2 - 1432\theta + 777 > 0 \) for all \( \theta \in [0, \frac{1}{2}] \) and \( n \geq 1 \).

\footnote{Our numerical simulation suggests that the qualitative nature of the result would remain unchanged for all \( n \geq 3 \).}
6 Price competition: Appendix to footnote 19 in Section 6

Consider a differentiated oligopoly with 3 firms where each firm $i$ produces a brand $i$ at unit cost $c_i$. Assume that $c_1 = c - x$, $c_3 = c$ and $c_2 = c - kx$ where $k = 1$ if firm 1 transfers knowledge to firm 2 and $k = 0$ otherwise. Inverse demand curve for brand $i$ is given by

$$p_i = a - q_i - b(q_j + q_k),$$

and the corresponding direct demand is:

$$q_i = \frac{a(1 - b) - (1 + b)p_i + b(p_j + p_k)}{(1 - b)(1 + 2b)} = D_i(p),$$

where $i, j, k \in \{1, 2, 3\}$ and $i \neq j \neq k$ and $p = (p_1, p_2, p_3)$.

As in the main text, firms 1 and 2 negotiate and jointly choose the level of firm 1’s ownership in firm 2’s equity, denoted $\theta$ ($0 \leq \theta \leq 1$). Subsequently in stage 2 firm 1 decides whether to transfer knowledge or not and finally in stage 3 firms engage in product market competition. The key difference here is that firms choose prices rather than quantities.

We focus on $\theta \in [0, \frac{1}{2}]$ where each firm $i$ simultaneously and non-cooperatively chooses $p_i$ to maximize its profit. If $\theta \in (\frac{1}{2}, 1]$, firm 1 chooses $q_1$ and $q_2$ and firm $m = 3, ..., n + 2$ chooses $q_m$, simultaneously and non-cooperatively, to maximize its own profit. For a given $\theta$ profits are as follows:

$$\pi_1(\theta, k, p) = [p_1 - (c - x)]D_1(p) + \theta[p_2 - (c - kx)]D_2(p),$$
$$\pi_2(\theta, k, p) = (1 - \theta)[p_2 - (c - kx)]D_2(p),$$
$$\pi_3(\theta, k, p) = [p_3 - c]D_3(p).$$

Let $p_i^*(\theta, k)$ and $\pi_i^*(\theta, k)$ respectively denote firm $i$’s quantity and profit in the equilibrium of the Stage 3 subgame represented by $(\theta, k)$. Define

$$\pi_{12}^*(\theta, k) = \pi_1(\theta, k, p^*) + \pi_2(\theta, k, p^*),$$

where $p^* = (p_1^*, p_2^*, p_3^*)$. We find that

$$\frac{d\pi_{12}^*(\theta, 1)}{d\theta} = \frac{[(2 + b - 3b^2)(a - c) + 2x(1 + b - b^2)][4(1 - \theta) + 2b(7 - 4\theta) + b^2(12 - \theta)]}{(1 - b)(1 + 2b)(2 + 3b)(4 + 6b - \theta b^2)^3} > 0$$

and

$$\frac{d\pi_{12}^*(\theta, 0)}{d\theta} = \frac{b^2[(2 + b - 3b^2)(a - c) - b(1 + b)x][A + Bx]}{(1 - b)(1 + 2b)(4 + 6b - \theta b^2)^3} > 0,$$

where $A = (a-c)(1-b)(4(1-\theta)+2b(7-4\theta)+b^2(1+\theta))$ and $B = b((2+2b-4b^2)(1+\theta)+3b^2(1+\theta))$.

Both $\pi_{12}^*(\theta, 1)$ and $\pi_{12}^*(\theta, 0)$ are increasing in $\theta$. As a consequence, no $\theta < \frac{1}{2}$ will arise as equilibrium outcome even if minimum PEO required to knowledge transfer is less than $\frac{1}{2}$.
7 PEO, consumer surplus and welfare when all firms have different costs

In this section, we consider what would happen with policy implications of our model if all firms had different costs and any combinations of the two firms can be alliance partners of the PEO arrangement. To this end, we have considered the effects of mergers and PEOs on consumer surplus under a set up in which all the $n+2$ firms have different costs. Without loss of generality assume that $c_1 < c_2 < \ldots < c_{n+2}$.

First consider a merger between firm $u$ and firm $v(> u)$. Equilibrium output under merger between firms $u$ and $v$ is implicitly given by the value of $Q$ that solves the following equation:

$$(n + 1)P(Q) + QP'(Q) = \sum c_i - c_v.$$  

Given that $(n + 2 - \theta)P(Q) + QP'(Q)$ is strictly decreasing in $Q$ it follows that aggregate output and consumer surplus is increasing in $c_v$. For any firm $u$, the higher the cost of its merger partner firm $v(> u)$ the higher is the consumer surplus. Thus, among all bilateral mergers consumer surplus is highest when $v = n + 2$.

Let us now turn from merger to PEO. Suppose firm $u$ can form a PEO alliance with firm $v$ but there is no knowledge transfer. Let $\theta$ denote the level of firm $u$’s ownership in firm $v$’s equity. Equilibrium aggregate output for such a PEO arrangement is given by $Q$ that solves the following equation:

$$(n + 2 - \theta)P(Q) + QP'(Q) = \sum c_i - \theta c_v.$$  

As in the case of merger, for any firm $u$, the higher the cost of its alliance partner $v(> u)$ the higher is consumer surplus. Thus, among all bilateral PEO arrangements where a firm $u$ owns a PEO of $\theta$ in firm $v$, consumer surplus is highest when $v = n + 2$.

Now consider PEO with knowledge transfer. Assume that firm $u$ owns a give level of PEO $\theta$ in firm $v$ and firm $v$’s unit cost is automatically reduced to $c_u$. It is straightforward to show that under such PEO arrangement with knowledge transfer, equilibrium aggregate output implicitly solves the following:

$$(n + 2 - \theta)P(Q) + QP'(Q) = \sum c_i + (1 - \theta)c_u - c_v.$$  

Since $LHS$ of the above equation is strictly decreasing in $Q$, aggregate output is highest when $u = 1$ and $v = n + 2$. In other words, for a given $\theta$ that induce knowledge transfer between all pairs, $CS$ is highest when firm 1 transfers knowledge to firm $n + 2$.

What would happen when there is a PEO between a firm with just average costs and one with higher than average costs? To answer that rearrange (3) as follows:

$$(n + 2 - \theta)P(Q) + QP'(Q) = \sum c_i - \theta c_v - (1 - \theta)(c_v - c_u)$$  

Since $LHS$ of the above equation is strictly decreasing in $Q$, aggregate output is highest when $u = 1$ and $v = n + 2$. In other words, for a given $\theta$ that induce knowledge transfer between all pairs, $CS$ is highest when firm 1 transfers knowledge to firm $n + 2$. 

What would happen when there is a PEO between a firm with just average costs and one with higher than average costs? To answer that rearrange (3) as follows:
Assume there are 3 firms where firm 2’s unit cost equals average unit cost, i.e. $c_2 - c_1 = c_3 - c_2$. For this cost specification, observe that RHS is lower for $v = 3$ compared to $v = 2$. Given $(n + 2 - \theta)P(Q) + QP'(Q)$ is strictly decreasing in $Q$ it follows that aggregate output and consumer surplus is higher when firm 2 forms a PEO with firm 3.

Although we have a clear cut result in case of exogenous $\theta$ with knowledge transfer, there is one critical complication in our model. The level of PEO is endogenously determined to induce a technologically superior firm’s knowledge transfer at the minimum PEO for knowledge transfer. To see the complication, consider a PEO alliance between firm $u$ and firm $v$ ($u < v$). The greater the $v$, the larger is the extent of the cost reduction $c_v - c_u$ due to knowledge transfer. This suggests consumer surplus is increasing in $c_v$. However, typically higher $c_v - c_u$ will also require higher level of minimum PEO $\theta$ (for inducing knowledge transfer) which suggests consumer surplus is decreasing in $c_v$. Thus, even for a given $u$, optimal $v$ from the standpoint of maximizing consumer surplus is not obvious and would depend on parameterizations. Thus in general consumer surplus ranking of PEO arrangements is ambiguous. We were unable to find out a tractable way to identify the optimal combinations of $u$ and $v$ when the PEO is endogenously determined.